# Spectrogram Inversion for Audio Source Separation via Consistency, Mixing, and Magnitude Constraints 

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# Introduction 

## Audio source separation

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## Framework

$\triangleright$ Monaural signals.
$\triangleright$ Short-time Fourier transform (STFT)-domain separation.
$\triangleright$ Mixture model: $\mathbf{X}=\sum_{j=1}^{J} \mathbf{S}_{j}$.


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$\triangleright$ The mixture's phase is assigned to each source using a Wiener-like filter or masking process. $x$ Issues in sound quality when sources overlap in the TF domain.

## Phase recovery for source separation



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$\checkmark$ State-of-the-art results, alleviate the phase issue.
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## Optimization-based algorithms

$\triangleright$ Preserves the magnitude/phase structure.
$\triangleright$ Allow for time-domain training through deep unfolding.
$\triangleright$ Can be combined with deep phase priors as initialization.

## Spectrogram inversion algorithms

Key ingredients to derive such algorithms:
$\triangleright$ Important properties in the STFT domain.
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## Mixing

[Wisdom, 2019]
[Wisdom, 2019]
[Wang, 2019]
[Gunawan, 2010]
[Wisdom, 2019]
[Le Roux, 2013]
Consistency
unawan, 2010]

Magnitude

## Proposal

A general framework for deriving spectrogram inversion algorithms based on these STFT constraints.

## Proposed framework

## Overview

## Proposed framework

$\triangleright$ For each property/objective/constraint, define a loss function (and an auxiliary function).
$\triangleright$ Combine them (soft penalties / hard constraints) to formulate optimization problems.
$\triangleright$ Derive algorithms that alternate projections on the corresponding constraints subspaces.

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## Auxiliary function method

$\triangleright$ Considering minimization of $\phi$, construct $\phi^{+}$such that:

$$
\phi(\theta)=\min _{\tilde{\theta}} \phi^{+}(\theta, \tilde{\theta}) .
$$

$\triangleright \phi$ is non-increasing when minimizing $\phi^{+}$with respect to $\theta$ and $\tilde{\theta}$ alternately.
$\checkmark$ Convergence, successfully used in audio, no hyperparameter to tune.


## STFT-domain constraints


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$\triangleright$ Consistency: the estimates (=complex-valued matrices) should be the STFT of time-domain signals.
$\triangleright$ Magnitude match: the estimates' magnitude should remain close to the output of the DNN computed beforehand.

## Mixing constraint

Loss function that promotes conservative estimates:

$$
\text { Complex-valued matrices } \mathbb{C}^{F \times T}
$$

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h(\mathbf{S})=\left\|\mathbf{X}-\sum_{j} \mathbf{S}_{j}\right\|^{2}
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## Auxiliary function

$\triangleright$ Auxiliary parameters $\mathbf{Y}$ such that $\sum_{j} \mathbf{Y}_{j}=\mathbf{X}$.
$\triangleright$ Positive weights $\boldsymbol{\Lambda}_{j}$ such that $\sum_{j} \lambda_{j, f, t}=1$.
$\triangleright$ Then the following is an auxiliary function for $h$ :

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h^{+}(\mathbf{S}, \mathbf{Y})=\sum_{j, f, t} \frac{\left|y_{j, f, t}-s_{j, f, t}\right|^{2}}{\lambda_{j, f, t}}
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$\triangleright$ Defines a projector $\mathcal{P}_{\text {mix }}$ onto the subspace of matrices complying with the mixing constraint.

## Consistency constraint

## Complex-valued matrices $\mathbb{C}^{F \times T}$

Loss function that promotes consistent estimates:

$$
i(\mathbf{S})=\sum_{j}\left\|\mathbf{S}_{j}-\mathcal{G}\left(\mathbf{S}_{j}\right)\right\|^{2} \text { with } \mathcal{G}=\text { STFT } \circ \text { iSTFT }
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$\triangleright$ Defines a projector $\mathcal{P}_{\text {cons }}$ onto the subspace of consistent matrices.

## Magnitude constraint

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Loss function that ensures the estimates' magnitudes remain close to the target value $\mathbf{V}_{j}$ estimated beforehand (e.g., using a DNN):

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$\triangleright$ Defines a projector $\mathcal{P}_{\text {mag }}$ onto the subspace of matrices whose magnitude equals the target value.

## Algorithm derivation example: problem setting

Main problem: optimize the mixing objective + soft consistency penalty + hard magnitude constraint (call that Mix+Incons_hardMag).

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\min _{\mathbf{S}} h(\mathbf{S})+\sigma i(\mathbf{S}) \text { such that }\left|\mathbf{S}_{j}\right|=\mathbf{V}_{j}
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Using our auxiliary function framework, this rewrites:

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\min _{\mathbf{S}, \mathbf{Y}, \mathbf{Z}} h^{+}(\mathbf{S}, \mathbf{Y})+\sigma i^{+}(\mathbf{S}, \mathbf{Z}) \text { such that }\left\{\begin{array}{l}
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$\triangleright$ Auxiliary parameters updates ( $\mathbf{Y}$ and $\mathbf{Z}$ ) are already known.
$\triangleright$ So let's focus on the update on $\mathbf{S}$.

## Algorithm derivation example: update

## New problem

$\triangleright$ Incorporate the hard constraint using the method of Lagrange multipliers.
$\triangleright$ Find a critical point for:

$$
h^{+}(\mathbf{S}, \mathbf{Y})+\sigma i^{+}(\mathbf{S}, \mathbf{Z})+\sum_{j, f, t} \delta_{j, f, t}\left(\left|s_{j, f, t}\right|^{2}-v_{j, f, t}^{2}\right)
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## Update

$\triangleright$ Set the partial derivative with respect to $\mathbf{S}$ at 0 and solve:

$$
\mathbf{S}_{j}=\frac{\mathbf{Y}_{j}+\sigma \boldsymbol{\Lambda}_{j} \odot \mathbf{Z}_{j}}{\left|\mathbf{Y}_{j}+\sigma \boldsymbol{\Lambda}_{j} \odot \mathbf{Z}_{j}\right|} \odot \mathbf{V}_{j}
$$

$\triangleright$ Generalizes particular cases from the literature ( $\sigma=0$ and $\sigma=+\infty$ ).

## Algorithm derivation example: illustration

Compact update rule using the projectors: $\mathcal{P}_{\text {mag }}\left(\mathcal{P}_{\text {mix }}(\mathbf{S})+\sigma \boldsymbol{\Lambda} \odot \mathcal{P}_{\text {cons }}(\mathbf{S})\right)$

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| :--- | :--- |
| Mix+Incons | $\frac{1}{1+\sigma \boldsymbol{\Lambda}} \odot\left(\mathcal{P}_{\text {mix }}(\mathbf{S})+\sigma \mathbf{\Lambda} \odot \mathcal{P}_{\text {cons }}(\mathbf{S})\right)$ |
| Mix+Incons_hardMag | $\mathcal{P}_{\text {mag }}\left(\mathcal{P}_{\text {mix }}(\mathbf{S})+\sigma \mathbf{\Lambda} \odot \mathcal{P}_{\text {cons }}(\mathbf{S})\right)$ |
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Consistency

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Some problem formulations / algorithms are not reported: ill-posed (conflicting constraints), impractical (2 redundant soft penalties), updates that only affect magnitude...

## Experiments

## Protocol

Task: speech enhancement ( $J=2$ )
$\triangleright$ Clean speech (VoiceBank) + noise (DEMAND: living room, bus, and public square noises).
$\triangleright$ Mixtures at various input SNR (iSNR): $-10,0$, and 10 dB .
$\triangleright 100$ mixtures (50/50 for validation/test).

## Magnitude estimation

$\triangleright$ Open-Unmix: a freely available BLSTM network (trained on different speakers and noises).
$\triangleright$ In practice, magnitudes are estimated more accurately as the iSNR increases.

## Methods

$\triangleright$ Initialization with an amplitude mask $(A M)=$ estimated magnitude + mixture's phase.
$\triangleright$ MISI is a widely-used baseline algorithm.
$\triangleright$ The consistency weight $\sigma$ and number of iterations are tuned on the validation set.
Separation quality measured with the speech signal-to-distortion ratio (SDR).

## Validation results



## Consistency weight

$\triangleright$ SDR peak: adjusting $\sigma$ maximizes the performance.
$\triangleright$ Our general framework $>$ particular cases $(\sigma=0$ or $+\infty)$ corresponding to existing algorithms.

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## Iterations

$\triangleright$ MISI reaches its peak performance after very few iterations.
$\triangleright$ Alternative algorithms are more stable / easier to tune.
$\triangleright$ For a fair comparison, use an algorithm-specific number of iterations (often overlooked).

## Test results

|  | iSNR $=10 \mathrm{~dB}$ | iSNR $=0 \mathrm{~dB}$ | iSNR $=-10 \mathrm{~dB}$ |
| :--- | :---: | :---: | :---: |
| AM | 18.7 | 13.5 | 7.7 |
| MISI | $\mathbf{1 9 . 6}$ | $\mathbf{1 4 . 1}$ | 7.7 |

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| Mix+Incons_hardMag | 18.7 | 13.8 | $\mathbf{7 . 9}$ |

$\triangleright$ Mag+Incons_hardMix: interesting alternative to MISI (same performance, stable over iterations).
$\triangleright$ Incons_hardMix: the performance degrades as the iSNR decreases.
$\triangleright$ Mix+Incons_hardMag $>$ MISI at low iSNR, but not at high iSNR ( $\neq$ from previous studies: optimized number of iterations and different magnitude estimation technique).

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| Mag+Incons_hardMix | $\mathbf{1 9 . 6}$ | $\mathbf{1 4 . 1}$ | 7.7 |
| Incons_hardMix | $\mathbf{1 9 . 6}$ | 13.9 | 7.5 |
| Mix+Incons_hardMag | 18.7 | 13.8 | 7.9 |
| Mix+Incons | 19.3 | 13.7 | $\mathbf{8 . 1}$ |

$\triangleright$ Mag+Incons_hardMix: interesting alternative to MISI (same performance, stable over iterations).
$\triangleright$ Incons_hardMix: the performance degrades as the iSNR decreases.
$\triangleright$ Mix+Incons_hardMag $>$ MISI at low iSNR, but not at high iSNR ( $\neq$ from previous studies: optimized number of iterations and different magnitude estimation technique).
$\triangleright$ Mix+Incons: mitigates the SDR drop at high iSNR + boosts the performance at low iSNR.

## Conclusion

## Main contribution

A general framework for deriving spectrogram inversion algorithms for source separation.
$\triangleright$ Encompasses many existing techniques from the literature.
$\triangleright$ Some novel algorithms are interesting alternatives.
( J https://github.com/magronp/spectrogram-inversion

Future research / work in progress:
$\triangleright$ Unfold these algorithms into neural networks for time-domain separation.
$\triangleright$ Combine them with deep phase priors.
$\triangleright$ Application to music / speech separation.

