Signal Inpainting from Fourier Magnitudes

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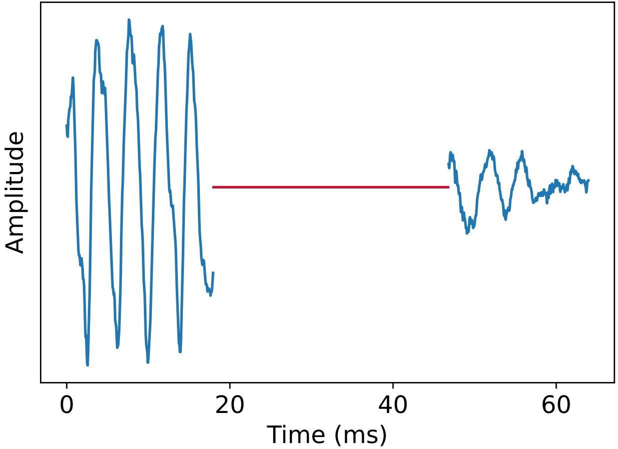
Problem formulation

Signal Inpainting

- Restore missing samples from a signal: *inpainting* [Adler et al., 2012].



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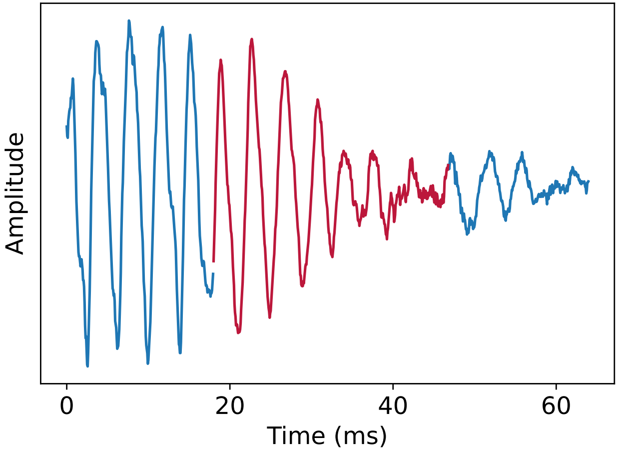

Signal Inpainting

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- Causes of degradation:
  - > Packet loss during transmission
  - > Digitalization of physically degraded media
  - > Degradation (clipping or impulsive noise).
- The location of the degraded samples is known.
- State of the art:
  - > Autoregressive models [Janssen et al., 1986]
  - > Sparse models [Mokry et al., 2019].

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Proposal: inpainting in the Fourier domain, where the problem is divided into two parts:

1. Restoring the Fourier magnitudes
2. Reconstructing the phases to inpaint the signal in time-domain.

Existing techniques to restore the Fourier magnitudes:

- Nonnegative Matrix Fatorization [Févotte et al., 2009]
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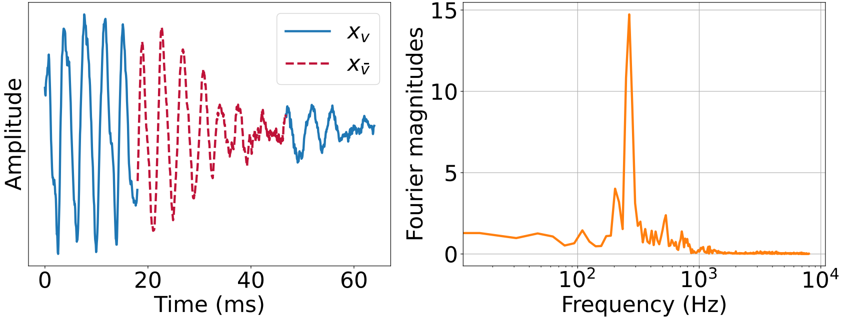
Phase retrieval with inpainting constraint



We assume the Fourier magnitudes $\boldsymbol{b} \in \mathbb{R}_+^L$ are observed (oracle or estimated).
$\boldsymbol{\Phi} \in \mathbb{C}^{L \times L}$: Discrete Fourier Transform matrix;

$$\underset{\boldsymbol{x} \in \mathbb{R}^L}{\text{minimize}} \, \| |\boldsymbol{\Phi x}| - \boldsymbol{b} \|^2 \quad \text{s.t.} \quad \boldsymbol{x}_v = \boldsymbol{x}_v^\natural$$

Difference with the classical phase retrieval problem

Classical phase retrieval problem:

$$\underset{\boldsymbol{x}\in\mathbb{R}^L}{\text{minimize}} \left\| |\boldsymbol{\Phi}\boldsymbol{x}| - \boldsymbol{b} \right\|^2 \quad \text{s.t.} \quad x_v = x_v^\natural$$

State of the art:

- Alternating minimization [Gerchberg and Saxton, 1972]
- Convex relaxation [Waldspurger et al., 2015]
- Gradient descent [Candès et al., 2015].

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Overview of the methods

Our problem:

$$\underset{\boldsymbol{x}\in\mathbb{R}^L}{\text{minimize}} \left\| |\boldsymbol{\Phi x}| - \boldsymbol{b} \right\|^2 \quad \text{s.t.} \quad \boldsymbol{x}_v = \boldsymbol{x}_v^\natural$$

Developed methods:

- Alternating minimization (AM)
- Convex relaxation (CR).

Methods

Alternating minimization: Formulation

$$\underset{\boldsymbol{x}\in\mathbb{R}^L}{\text{minimize}} \left\| |\boldsymbol{\Phi}\boldsymbol{x}| - \boldsymbol{b} \right\|^2 \quad \text{s.t.} \quad \boldsymbol{x}_v = \boldsymbol{x}_v^\natural$$

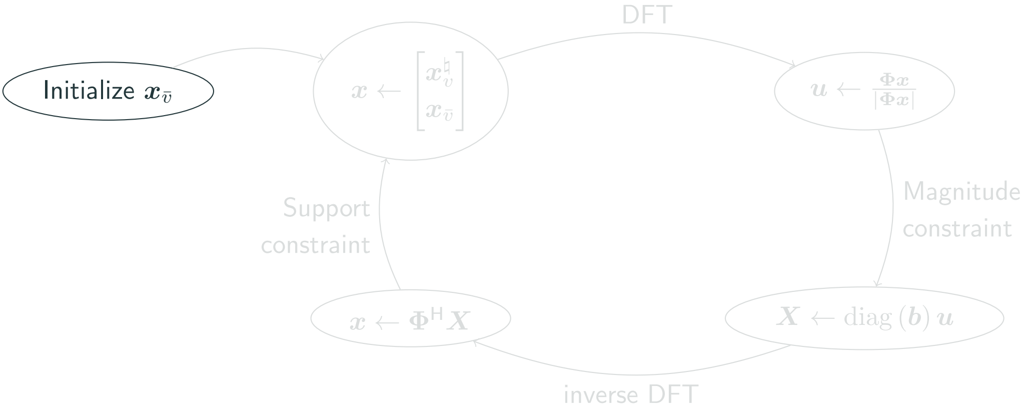
Introducing an auxiliary phase variable $\boldsymbol{u}$, this problem is equivalent to:

$$\underset{\boldsymbol{x}\in\mathbb{R}^L, \boldsymbol{u}\in\mathbb{C}^L}{\text{minimize}} \left\| \boldsymbol{\Phi}\boldsymbol{x} - \text{diag}(\boldsymbol{b})\boldsymbol{u} \right\|^2 \quad \text{s.t.} \quad \boldsymbol{x}_v = \boldsymbol{x}_v^\natural \quad \text{and} \quad |\boldsymbol{u}| = 1.$$

$\text{diag}(\boldsymbol{b})$: square matrix whose diagonal is $\boldsymbol{b}$

We reorder $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_v \\ \boldsymbol{x}_{\bar{v}} \end{bmatrix}$ and $\boldsymbol{\Phi} = \underbrace{[\boldsymbol{\Phi}_v, \boldsymbol{\Phi}_{\bar{v}}]}_{\text{Column-wise reordering}}$.

Alternating minimization: Algorithm



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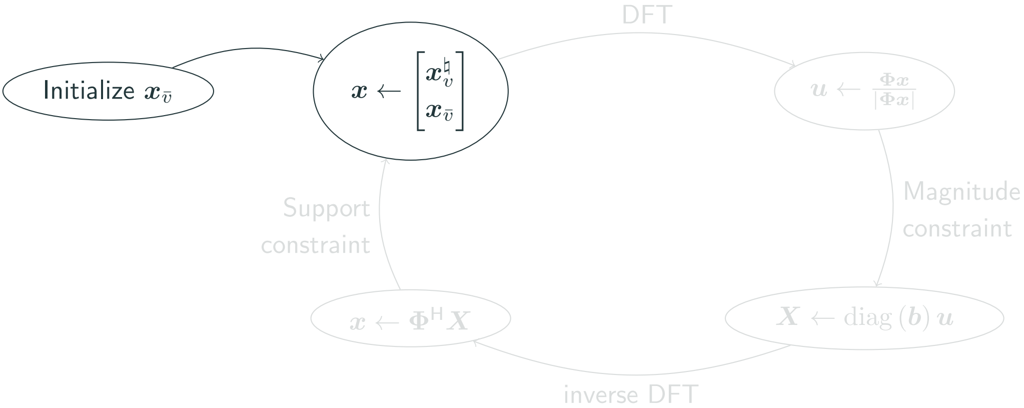

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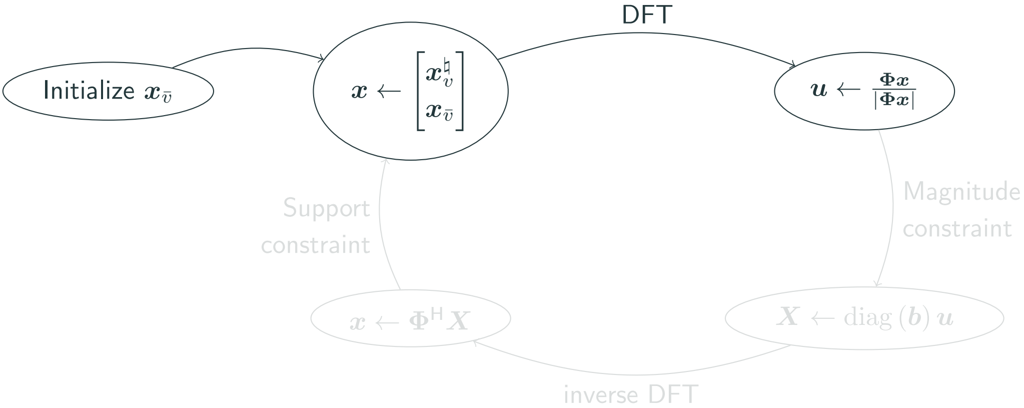

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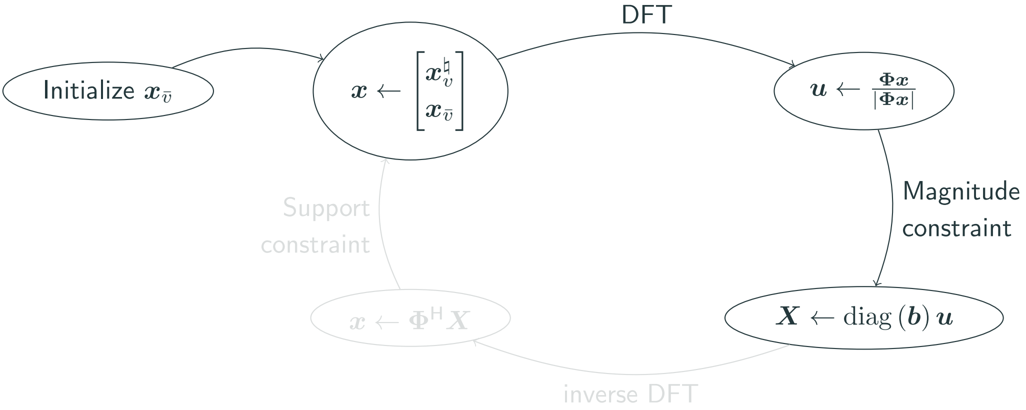

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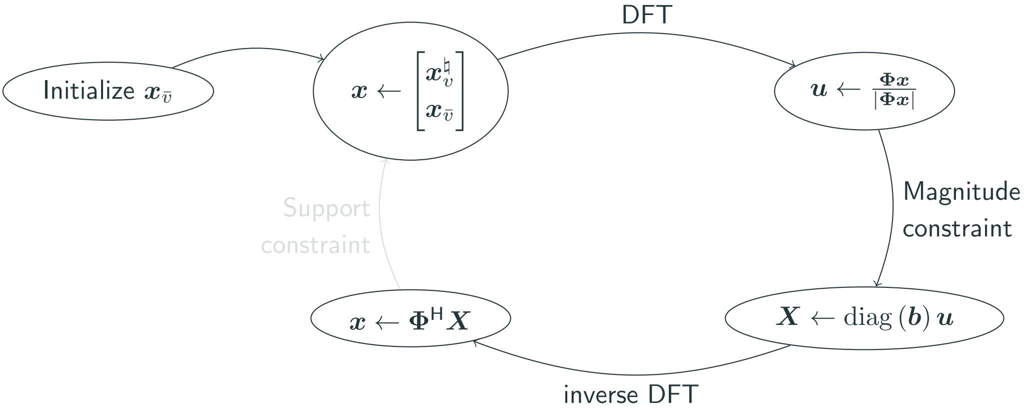

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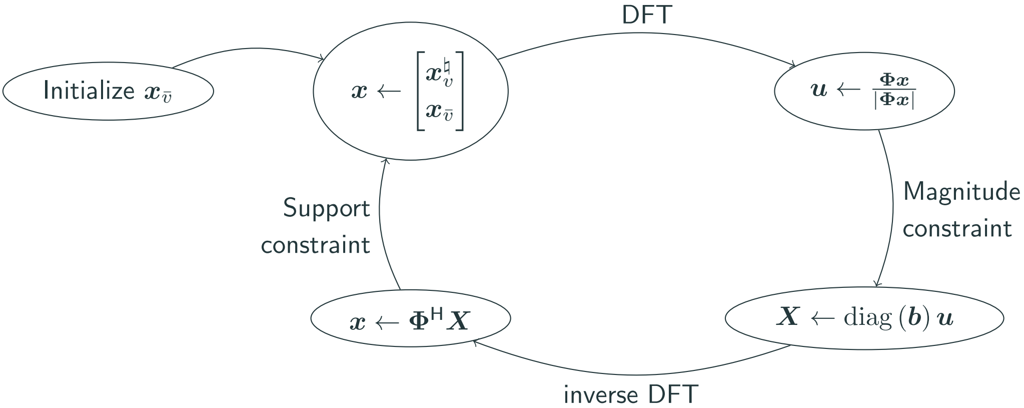

Convex Relaxation: Formulation

Considering the previously reformulated problem:

$$\underset{\boldsymbol{x}\in\mathbb{R}^L,\boldsymbol{u}\in\mathbb{C}^L}{\text{minimize}} \left\|\boldsymbol{\Phi}_v\boldsymbol{x}_v^\natural + \boldsymbol{\Phi}_{\bar{v}}\boldsymbol{x}_{\bar{v}} - \operatorname{diag}(\boldsymbol{b})\boldsymbol{u}\right\|^2 \quad \text{s.t.} \quad |\boldsymbol{u}| = 1,$$

and introducing:

$$\tilde{\boldsymbol{m}} := \left[\left(\boldsymbol{\Phi}_{\bar{v}}\boldsymbol{\Phi}_{\bar{v}}^{\mathsf{H}} - \boldsymbol{I}\right)\operatorname{diag}(\boldsymbol{b})\ ,\ \boldsymbol{\Phi}_v\boldsymbol{x}_v\right] \text{ and } \tilde{\boldsymbol{u}} = \begin{bmatrix}\boldsymbol{u}\\ 1\end{bmatrix},$$

we get the equivalent problem:

$$\underset{\tilde{\boldsymbol{u}}\in\mathbb{C}^{L+1}}{\text{minimize}} \left\|\tilde{\boldsymbol{m}}\tilde{\boldsymbol{u}}\right\|^2 \quad \text{s.t.} \quad |\tilde{\boldsymbol{u}}| = 1 \text{ and } \tilde{\boldsymbol{u}}[L] = 1$$

Convex Relaxation: Lifting

$$\underset{\tilde{\boldsymbol{u}} \in \mathbb{C}^{L+1}}{\text{minimize}} \left\| \tilde{\boldsymbol{m}} \tilde{\boldsymbol{u}} \right\|^2 \quad \text{s.t.} \quad |\tilde{\boldsymbol{u}}| = 1 \text{ and } \tilde{\boldsymbol{u}}[L] = 1$$

We lift and relax the problem as in [Waldspurger et al., 2015], by setting:

$$\tilde{\boldsymbol{U}} = \tilde{\boldsymbol{u}} \tilde{\boldsymbol{u}}^{\mathsf{H}}, \quad \tilde{\boldsymbol{M}} = \tilde{\boldsymbol{m}}^{\mathsf{H}} \tilde{\boldsymbol{m}}, \quad \text{Rank}\left(\tilde{\boldsymbol{U}}\right) = 1.$$

The problem becomes:

$$\underset{\tilde{\boldsymbol{U}}}{\text{minimize}} \, \text{Tr}\left(\tilde{\boldsymbol{M}} \tilde{\boldsymbol{U}}\right) \text{ s.t. } \text{diag}\left(\tilde{\boldsymbol{U}}\right) = 1, \, \tilde{\boldsymbol{U}} \succeq 0 \text{ and } \text{Rank}\left(\tilde{\boldsymbol{U}}\right) = 1.$$

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Original problem: $\underset{\boldsymbol{x}\in\mathbb{R}^L}{\text{minimize}} \left\| |\Phi\boldsymbol{x}| - \boldsymbol{b}\right\|^2$ s.t. $\boldsymbol{x}_v = \boldsymbol{x}_v^\natural$

	Loss	Search space
Lifted problem	Convex	Non Convex
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→ We relax the rank constraint, and solve the resulting Semi-Definite Program by Block Coordinate Descent.

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Combining the CR and AM methods

1. Estimate the solution $\tilde{\boldsymbol{U}}$ of the CR method
2. Compute $\tilde{\boldsymbol{u}} =$ the eigenvector associated with the largest eigenvalue (in modulus) of $\tilde{\boldsymbol{U}}$
3. Extract $\boldsymbol{u} = \dfrac{\tilde{\boldsymbol{u}}[:L-1]}{\tilde{\boldsymbol{u}}[L]}$
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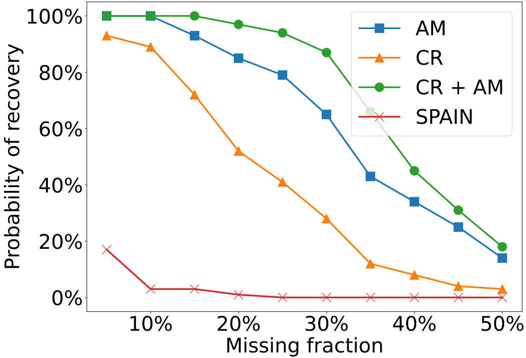
Experiments

Protocol

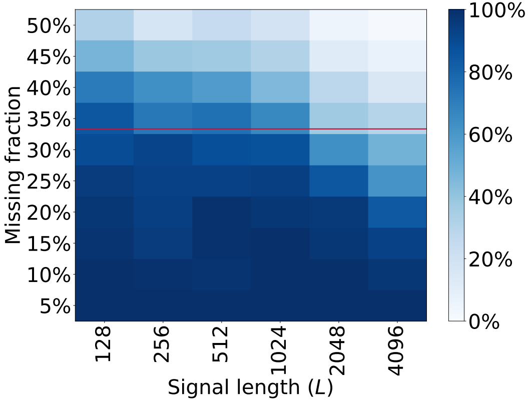
- Data: non-silent excerpts from Librispeech [Panayotov et al., 2015].
- Algorithms stopping criteria:
  - AM: 1000 iterations, or maximum loss improvement over the 5 last iterations $< 10^{-10}$
  - CR: 10 iterations.
- Reconstruction score: SER $= 10 \log_{10} \frac{\left\| \boldsymbol{x}_{\bar{v}}^{\natural} \right\|^2}{\left\| \boldsymbol{x}_{\bar{v}}^{\natural} - \boldsymbol{x}_{\bar{v}}^{*} \right\|^2}$, where $\boldsymbol{x}_{\bar{v}}^{*}$ is the prediction.
- Recovery if SER > 20 dB.
- Baseline: sparsity-based method SPAIN [Mokry et al., 2019].

Influence of the missing fraction onto performance

- 100 excerpts of length $L = 1024$ samples
- > All methods outperform SPAIN: They correctly leverage the Fourier magnitudes.
- > CR provides a solution that is more likely to converge to a global optimum after iterations of AM.



Influence of the signal length and missing fraction on the CR+AM method performance



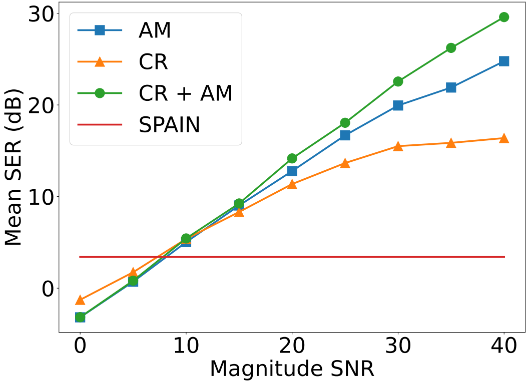
- CR+AM approach performs near the theoretical optimum when the missing fraction is lower than 33% and $L < 1024$.
- Performance decreases when L increases: Curse of dimensionality and non-convexity.

Influence of the magnitude noise

- $L = 1024$ samples, 25% missing fraction.
- Noisy magnitudes $\boldsymbol{b} = \max\left(0, \left|\boldsymbol{\Phi}\boldsymbol{x}^\natural\right| + \boldsymbol{n}\right)$, where $\boldsymbol{n}$ is a white Gaussian noise whose variance is adjusted to fit a given magnitude signal-to-noise ratio (SNR).

> Linear decay in performance on the log-log plot when SNR falls below 20 dB.

> Our methods outperform SPAIN at high SNRs only, hence the need to accurately estimate the magnitudes beforehand.



Audio example

- Setting:
  - > Audio split in windows of 64 ms $\leftrightarrow$ 1024 samples @16 kHz
  - > 45% missing.
- Results:
  - > Degraded audio
  - > Original audio
  - > Alternating Minimization, initialized with $\boldsymbol{x}_{\bar{v}} \leftarrow 0$: SER $= 18.4$ dB
  - > Convex Relaxation only SER $= 15.0$ dB
  - > Convex Relaxation as initialization for Alternated Minimization: SER $= 20.7$ dB

Conclusion

- Contributions:
  - A new formulation of the signal inpainting problem using Fourier magnitudes
  - Two methods based on Alternated Minimization and Convex Relaxation
  - Competitive results with state-of-the-art when magnitudes are accurately estimated.
- Perspectives:
  - Design magnitude restoration methods and combine them with AM/CR into a complete signal inpainting framework.

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