



# Towards complex nonnegative matrix factorization with the beta-divergence

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# Outline

- 1 Background and problem setting
- 2 Complex NMF
- 3 Experimental results



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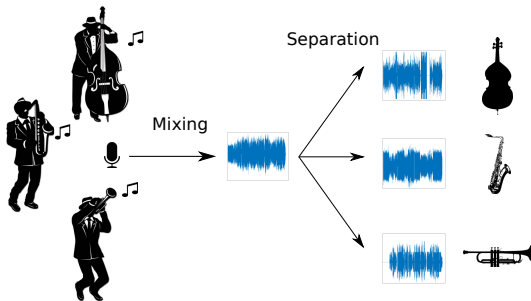
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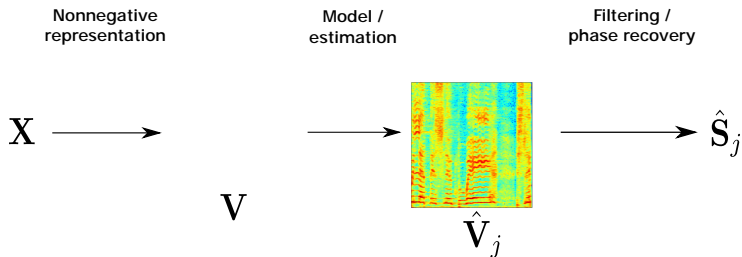
# Audio source separation



- Estimate constitutive sources that form a mixture;
- Applications: speech enhancement, augmented musical mixing...
- Challenges: Reduction of **interference** and **artifacts**.

# General framework

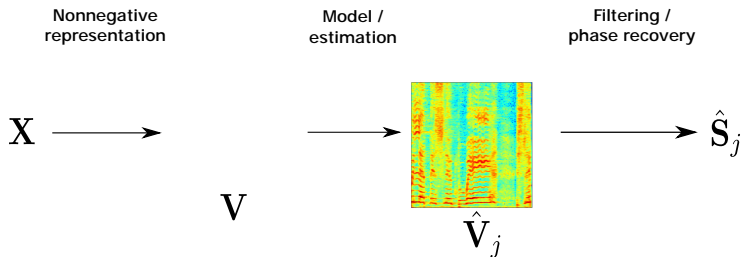
In the STFT domain:  $\mathbf{X} = \prod_j \mathbf{S}_j$ .



- Nonnegative representation: magnitude/power spectrogram;
- Spectrogram model: KAM, NMF, DNNs...
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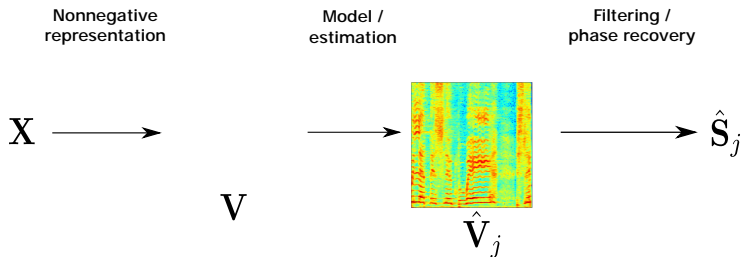
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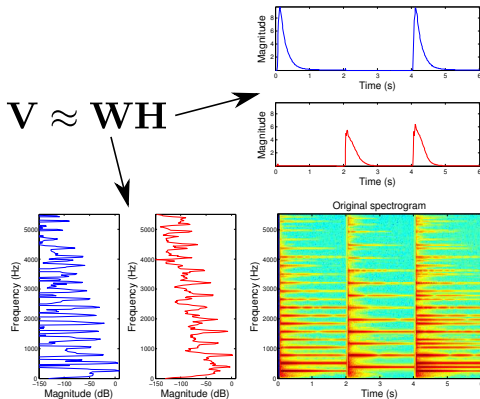
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# Nonnegative matrix factorization

Find a factorization of a nonnegative matrix  $\mathbf{V}$  (e.g., a spectrogram):





# NMF - beta-divergences

- Estimation of NMF:

$$\min_{W;H} D(V;WH)$$

- Popular choices for  $D$  are the beta-divergences  $D$  :

$\beta = 2$	$\beta = 1$	$\beta = 0$
Euclidean distance	Kullback-Leibler (KL) divergence	Itakura-Saito (IS) divergence
Emphasis on high-energy components	In between !	Scale invariance

- Optimization techniques (heuristic approach, majorize-minimization...) ! multiplicative updates rules.



# NMF - Wiener filtering

$$\hat{S}_j = \frac{W_j H_j}{\sum_{k=1}^J W_k H_k} X$$

Source STFT (Estimated)      Mask      Mixture STFT

- -source = -mixture.

Issues in sound quality when sources overlap in the TF domain:

Mixture    Original    Oracle Wiener



# Complex NMF

Directly models the complex-valued STFTs:

$$\hat{X} = \prod_{j=1}^J \underbrace{W_j H_j}_{\text{NMF magnitude}} \underbrace{e^{j_j}}_{\text{Phase}}$$

Estimated by minimizing  $\mathcal{D}_{\text{EUC}}(X; \hat{X})$ .

Allows to incorporate **phase constraints**;

Not straightforward to extend it to other beta-divergences.

! A probabilistic model in which complex NMF can be extended to any beta-divergence.

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# Gaussian model

Mixture in each TF bin:

$$x = \sum_{j=1}^J s_j$$

Gaussian sources:

$$s_j \sim \mathcal{N}(0; \sigma_j^2) \text{ with } \sigma_j^2 = \frac{\epsilon_j}{\eta_j}$$

- $\sigma_j^2$  = variance! energy of the source.
- $\epsilon_j$  = relation term! non-uniformity of the phase.

# Isotropic Gaussian sources

- Traditionally, circularly-symmetric (isotropic) sources:

$$\varphi_j = 0, \quad \text{uniform phase.}$$

- NMF variance:  $\sigma_j^2 = W_j H_j$ .

- Estimation:

$$\text{maximum likelihood} \quad \min D_{\text{IS}}(jX_j^2; WH)$$

! ISNMF model.

# Anisotropic Gaussian model

Here, anisotropic sources:

$$c_j \neq 0, \quad \text{non-uniform phase.}$$

The variance / relation terms are:

$$\begin{aligned} \sigma_j &= (1 - \alpha_j^2) W_j H_j \\ c_j &= W_j H_j e^{i 2 \phi_j} \end{aligned}$$

- $\phi_j$  = phase location parameter.
- $\alpha_j = \sigma_j / W_j H_j$  = functions of  $\phi_j$ , quantify the non-uniformity of the phase.
  - $\alpha_j = 0$  !  $\phi_j = [W_j H_j]$  and  $c_j = 0$  ! ISNMF



# Anisotropic Gaussian model

# Inference

The likelihood is not tractable    Expectation-maximization (EM).

- Maximize a lower bound  $Q$  of the likelihood.
- Alternate between:
  - E-step: compute  $Q$  given the current parameter estimates.
  - M-step: maximize  $Q$  to update the parameters.

# Phase-corrected posterior power

The EM functional rewrites:

$$Q \stackrel{c}{=} \sum_{j=1}^J \log(W_j H_j) + \frac{P_j}{W_j H_j} \stackrel{c}{=} \sum_{j=1}^J D_{\text{IS}}(P_j; W_j H_j)$$

with:

$$P_j / E_{\text{post}} \{s_j^H s_j\} > 0$$

- $P_j$  is phase-aware;
- $= 0 ! \quad P_j = E_{\text{post}} \{s_j^H s_j\} = \text{posterior power of } s_j .$

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# Introducing the beta-divergence

- Minimization of  $D_{IS}(P_j; W_j H_j)$  ! "Complex ISNMF".

Proposed approach: we replace the IS divergence with the beta-divergence.

- Heuristic! no more convergence guarantee.
- Minimization of  $D(P_j; W_j H_j)$  ! multiplicative updates.  
! "Complex NMF"

Phase-aware decomposition of the data;

Uses a distortion metric adapted to audio.

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# Setup

Harmonic/percussive source separation task ( $k=2$  sources).

Dataset:

- DSD100: 100 music songs, split into learning/test sets;
- Each excerpt is 20 seconds long.

Supervised separation scenario:

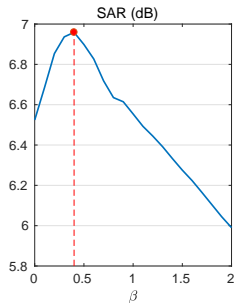
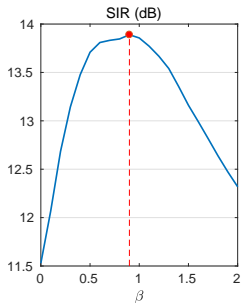
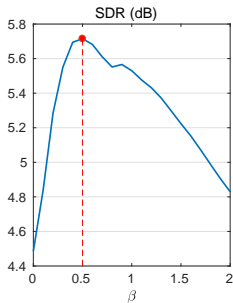
- Each excerpt is split into two signals of 10 seconds.
- The first is used for learning dictionaries  $W_j$  (k-means clustering with 50 basis per dictionary).
- The second is used for performing the separation.

Source separation quality:

- Signal-to-distortion/interference/artifact ratios (SDR, SIR, and SAR).

# Learning $\beta$

On the learning set:



- Trade-off between interference ( $\beta = 0.9$ ) and artifacts ( $\beta = 0.4$ ) reduction.
- $\beta = 0.5$  ! best overall distortion reduction.



# Comparison to other approaches

Baselines: NMF (also uses  $\alpha = 0.5$ ) and complex Euclidean NMF.

Median results over the test dataset:

	SDR	SIR	SAR
NMF	5:4	12:3	7:1
Complex Euclidean NMF	2:0	8:6	3:5
Complex NMF	<b>5.5</b>	<b>12.6</b>	<b>7.2</b>

- Complex NMF outperforms its Euclidean counterpart;
- Slightly better results than NMF.
  - ! Complex NMF allows to incorporate phase constraints.



# Conclusion

**A novel probabilistic framework where complex NMF can be extended to any beta-divergence.**

Future work:

- Better understanding of the phase-corrected posterior power;
- Phase-constrained complex NMF;
- Alternative probabilistic model / convergence guarantees;
- Use DNNs instead of NMF for joint magnitude/phase estimation.

*cf. some other IWAENC papers!*



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# Thanks!

 <https://github.com/magronp/complex-beta-nmf>

