TAMPERE UNIVERSITY OF TECHNOLOGY

Lévy NMF for robust nonnegative source separation

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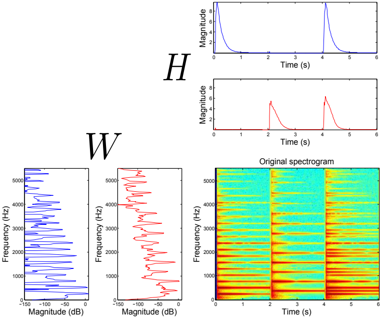
Source separation

- Problem : extract the X_k, $k \in \{1, ..., K\}$ from :

$$X = \sum_k X_k$$

- Nonnegative data : audio spectrograms, images, fluorescence spectra...
- Many methods : PCA, ICA, NMF...

Nonnegative matrix factorization



- Probabilistic approach : sources as latent variables ;
- Maximum likelihood $\leftrightarrow$ Minimization of a cost function between X and WH.

Robustness

Traditional distribution are not *heavy-tailed* : no robustness to outliers.

$\rightarrow$ Stable distributions :

- Stability and robustness...
- ... not a nonnegative support in general.

Goal : **A robust nonnegative data model for source separation**

Outline

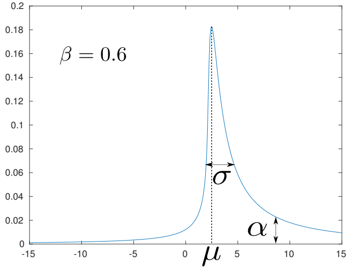
1 Lévy NMF model

2 Parameter estimation

3 Experimental evaluation

Stable distributions

A family of **heavy-tailed** distributions
Symmetric α-stable (SαS) : $\beta = 0$.
Stability : a sum of stable variables is stable.



Special cases :

- Gaussian : $\alpha = 2$ and $\beta = 0$;
- Cauchy : $\alpha = 1$ and $\beta = 0$;
- Lévy : $\alpha = 1/2$ and $\beta = 1$;

Positive α-stable distributions

In general, the support of the stable distributions is $\mathbb{R}$ (or $\mathbb{C}$).

For $\beta = 1$ and $\alpha < 1$, the support is $[\mu; +\infty[$.

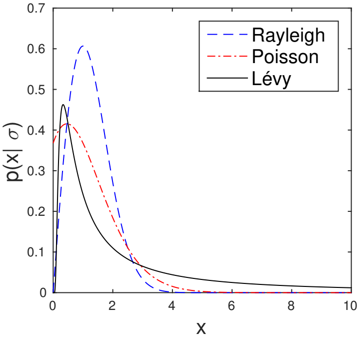
$\rightarrow$ **Positive α-stable** (PαS) distributions :

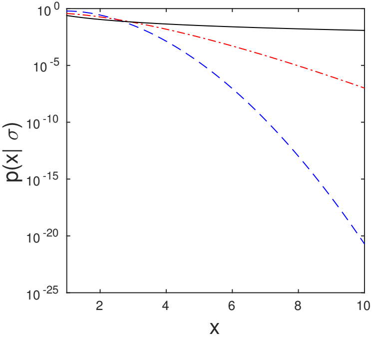
$$\mathcal{P}\alpha\mathcal{S}(\sigma) = \mathcal{S}(\alpha, 0, \sigma, 1), \text{ with } \alpha < 1.$$

Lévy distribution (α=1/2) :

$$p(x \mid \sigma) = \begin{cases} \sqrt{\frac{\sigma}{2\pi}} \frac{1}{x^{3/2}} e^{-\frac{\sigma}{2x}} & \text{if } x > 0 \\ 0 & \text{else.} \end{cases}$$

Positive α-stable distributions





Mixture model

- Nonnegative data $X \in \mathbb{R}_+^{F \times T} : X = \sum_k X_k$.
- Independent Lévy coefficients :

$$X_k(f,t) \sim \mathcal{L}(\sigma_k(f,t))$$

$$\rightarrow X \sim \mathcal{L}(\sigma) \text{ with } \sqrt{\sigma} = \sum_k \sqrt{\sigma_k}.$$

- NMF on the dispersion parameters :

$$\sqrt{\sigma} = WH,$$

where $W \in \mathbb{R}_+^{F \times K}$ and $H \in \mathbb{R}_+^{K \times T}$.

$\rightarrow$ **Lévy NMF** model.

Maximum likelihood (ML)

Log-likelihood of the data :

$$
\begin{aligned}
L(W,H) &= \sum_{f,t} \log(p(X(f,t);\sigma(f,t))) \\
&\overset{c}{=} \frac{1}{2} \sum_{f,t} \log([WH](f,t)^2) - \frac{[WH](f,t)^2}{X(f,t)} \\
&\overset{c}{=} -\frac{1}{2} d_{IS}([WH]^{\odot 2}, X),
\end{aligned}
$$

ML $\leftrightarrow$ Minimize the Itakura-Saito divergence between $[WH]^{\odot 2}$ and X

Heuristic approach

Decomposition of the gradient of $\mathcal{C}$ w.r.t. θ ($= W$ or H) :

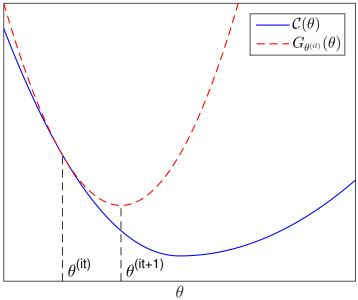
$$\frac{\partial \mathcal{C}}{\partial \theta} = \nabla_\theta^+ - \nabla_\theta^-, \text{ with } \nabla_\theta^+ > 0 \text{ and } \nabla_\theta^- > 0.$$

Update rule :

$$\theta \leftarrow \theta \odot \frac{\nabla_\theta^-}{\nabla_\theta^+}$$

- No guarantee that the cost function is non-increasing ;
- In practice, it is observed for many NMF models...
- ... but not for the Lévy case.

Majorize-Minimization



- Auxiliary function G :

$$\forall(\theta, \overline{\theta}),\ \mathcal{C}(\theta) \leq G_{\overline{\theta}}(\theta),\ \text{and}\ \mathcal{C}(\overline{\theta}) = G_{\overline{\theta}}(\overline{\theta})$$

- Update : $\theta^{(it+1)} = \arg\min_{\theta} G_{\theta^{(it)}}(\theta)$

Majorize-Minimization

- G is obtained by using convexity inequalities ;
- For Lévy NMF :

$$W \leftarrow W \odot \left(\frac{[WH]^{\odot -1} H^T}{([WH] \odot X^{\odot -1}) H^T} \right)^{\odot 1/2}$$

and

$$H \leftarrow H \odot \left(\frac{W^T [WH]^{\odot -1}}{W^T ([WH] \odot X^{\odot -1})} \right)^{\odot 1/2}$$

- Similar updates to the heuristic approach, with a power $1/2$.
- The cost function is non-increasing under these updates.

Lévy NMF vs. ISNMF

If $K = 1$ and $W(f) = 1 \; \forall f$:

$$H_{\text{IS}}(t) \leftarrow \frac{1}{F} \sum_f X(f,t), \; H_{\text{Lévy}}(t) \leftarrow \sqrt{\frac{F}{\sum_f \frac{1}{X(f,t)}}}.$$

- ISNMF $\rightarrow$ **Arithmetic** mean ;
- Lévy NMF $\rightarrow$ **Harmonic** mean (and $\sqrt{}$).

If $F = 10$ and $X(f,t) = 1$ except for one entry : $X(f_0, t_0) = 10^8$:

$$H_{\text{IS}}(t_0) \leftarrow 10^7, \; H_{\text{Lévy}}(t_0) \leftarrow 1.05.$$

$\rightarrow$ **Robustness of Lévy NMF**

Source estimation

- Natural estimator : $\hat{X}_k = \mathbb{E}_{X_k|X}(X_k)$.
- For **any** PαS distribution :

$$\hat{X}_k = \frac{{\sigma_k}^{\alpha}}{\sum_l {\sigma_l}^{\alpha}} \odot X \quad (1)$$

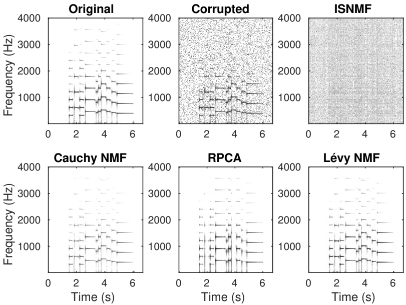
$\rightarrow$ **Generalized Wiener filtering**

- For Lévy NMF :

$$\hat{X}_k = \frac{W_k H_k}{\sum_l W_l H_l} \odot X \quad (2)$$

Music spectrogram inpainting

- Data : 6 guitar pieces ;
- The spectrograms are corrupted with impulsive noise ;
- The models are learned on the corrupted data ;
- The noise localization is unknown.



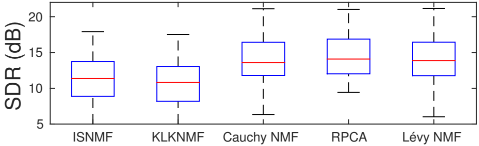
Music spectrogram inpainting

	Log(KL)	SDR (dB)	
ISNMF	9.0	−23.5	
KLNMF	6.2	−8.9	
Cauchy NMF	3.4	7.6	
RPCA	3.6	7.4	
Lévy NMF	**3.2**	**9.2**	
Weighted ISNMF	3.8	4.5	

- Bad results with classical NMFs (IS and KL) ;
- Lévy NMF compares with other methods.

Musical accompaniment enhancement

- Data : 50 music songs ;
- Musical accompaniment is assumed well-represented by a low-rank NMF model ;
- Voice is assumed to be similar to impulsive noise.



Mix Music IS Lévy

Conclusion

A robust nonnegative data model

- Many areas of application : data mining, applied physics...
- An extension of Wiener filtering to nonnegative data.

Future work

- MAP estimation : priors on the parameters (sparsity, temporal smoothness...) ;
- Generalization to PαS or inverse-Gamma distributions.