Sound synthesis of Siku and closed pipe flutes

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Introduction

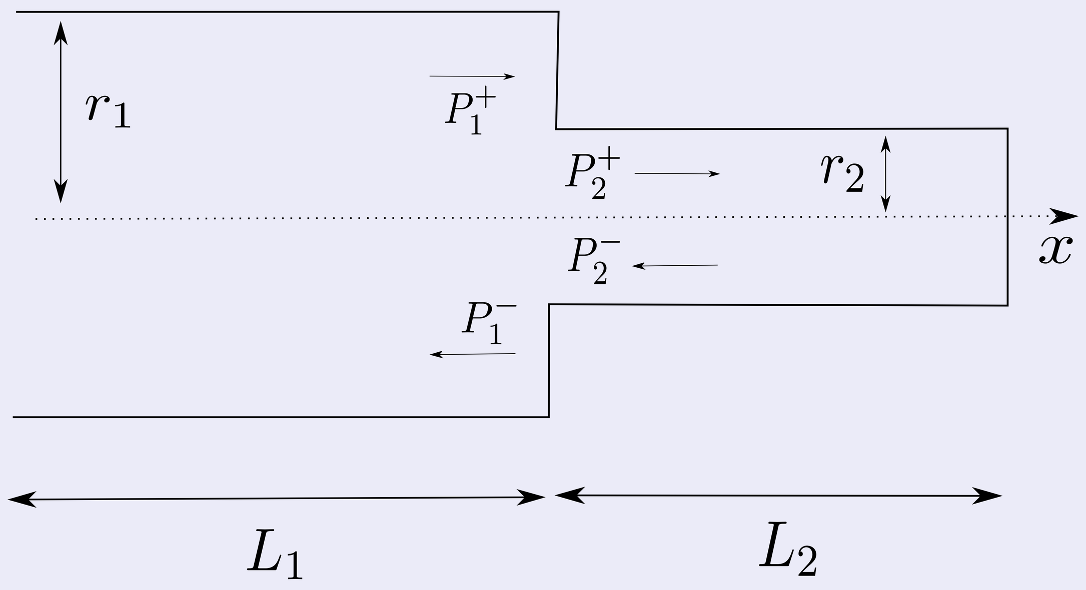
The siku

- Latin America pre-hispanic panpipe.
- Closed-end resonator made of two or three cylinders of different diameters.
- Loud and vibrant sound: *sonido rajado*.

Objectives

- Physics of the resonator.
- Links between geometry and resonances.
- Simulation model for sound synthesis.

Acoustic model



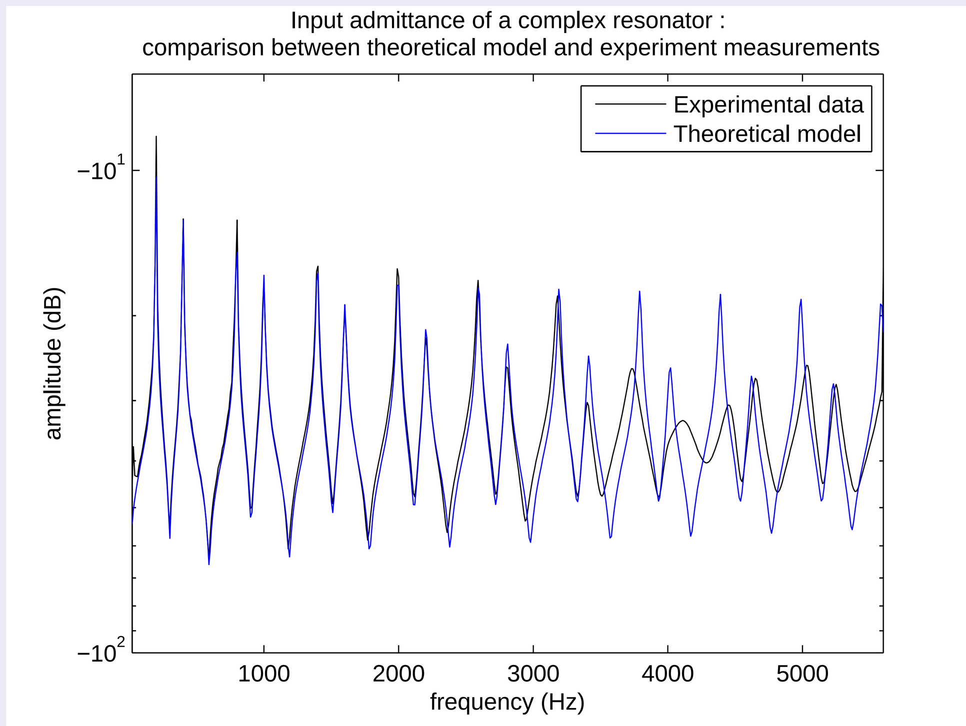
- Plane longitudinal waves
- Two-port junction
  - D'Alembert decomposition:

$$\begin{cases} P_2^+ = P_1^+ + r_k(P_1^+ - P_2^-) \\ P_1^- = P_2^- + r_k(P_1^+ - P_2^-) \end{cases} \quad r_k = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}.$$

- Visco-thermal losses
  - Complex wave number:

$$k = i\frac{\omega}{c} + (1+i)\frac{\beta}{r}\sqrt{f}.$$

- Input admittance



  - Double admittance maxima.
  - High accuracy model.
- Radiation
  - Circular plate model:

$$Z_{rad} = Z_c(\alpha_r(ka)^2 + i\alpha_i ka).$$

  - α_r set to $1/4$ and α_i variable (different instruments size).

Resonant frequencies

- Resonance condition
  - Input impedance is null:

$$\tan(\Im(k_1)L_1)\tan(\Im(k_2)L_2) = \frac{S_1}{S_2}$$

- Resonant frequencies without losses
  - Analytical resolution:

$$\tilde{f} = f_0 \times \begin{cases} 3n + \frac{3}{2}(1 - \frac{1}{\pi}\arccos r_k) \\ 3n + \frac{3}{2}(1 + \frac{1}{\pi}\arccos r_k) \end{cases} \quad n \in \mathbb{Z},\ f_0 = \frac{c}{6L}.$$

  - Two "almost" harmonic series.
  - A single harmonic series if

$$\frac{3}{2}(1 - \frac{1}{\pi}\arccos r_k) = 1.$$

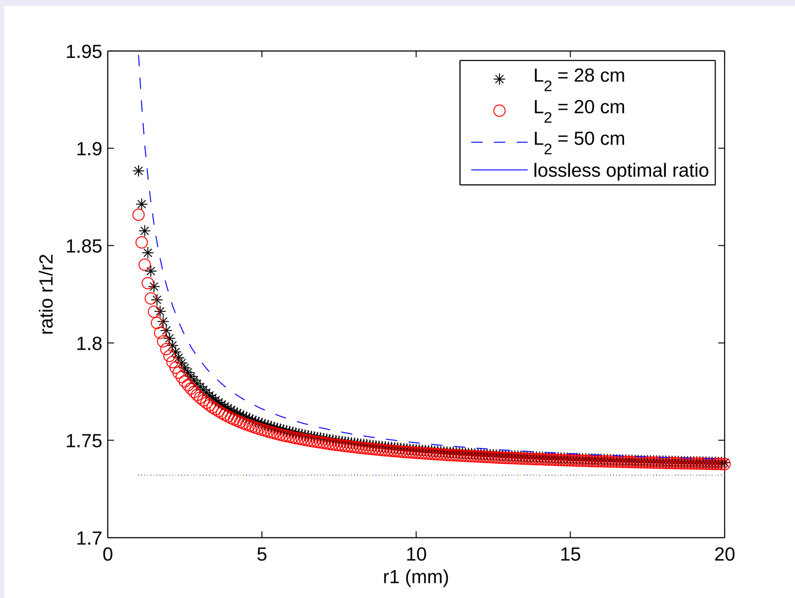
  - Lossless ratio between cylinders: $\sqrt{3}$.
- Resonant frequencies with losses
  - Approximation: standard geometry and frequency under **6000*Hz***:

$$f_{res} = \tilde{f} - b\sqrt{\tilde{f}},\ b = (\frac{L_1}{r_1} + \frac{L_2}{r_2})\frac{c\beta}{4\pi L}.$$

  - $f_{res} - \tilde{f}$ not constant: impossible to obtain perfect harmonicity.

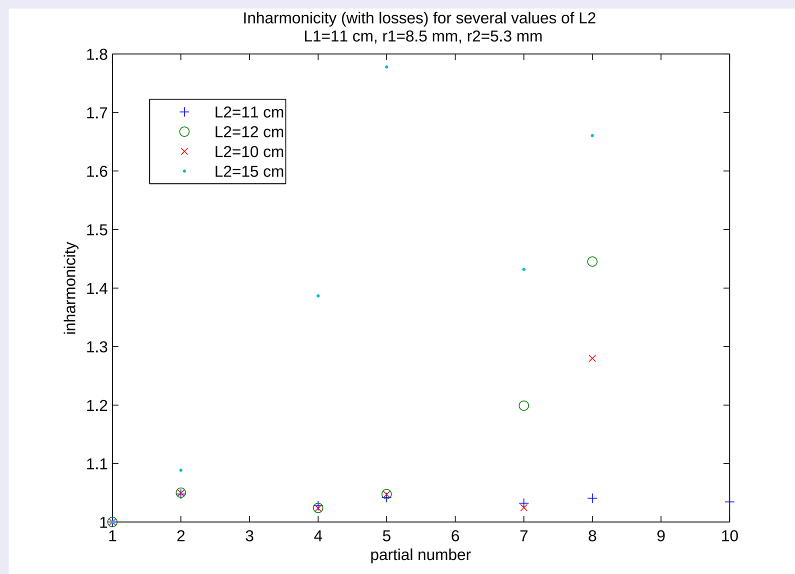
- Optimal ratio
  - Lossless ratio cancel harmonicity for the high frequencies.
  - Harmonicity condition for the first partials:

$$3x_k - 1 = \frac{\beta}{2\pi}\sqrt{\frac{c}{L}}(\frac{L_1}{r_1} + \frac{L_2}{r_2})(\sqrt{1+x_k} - 2\sqrt{1-x_k}) \text{ with } x_k = \frac{1}{\pi}\arccos r_k.$$

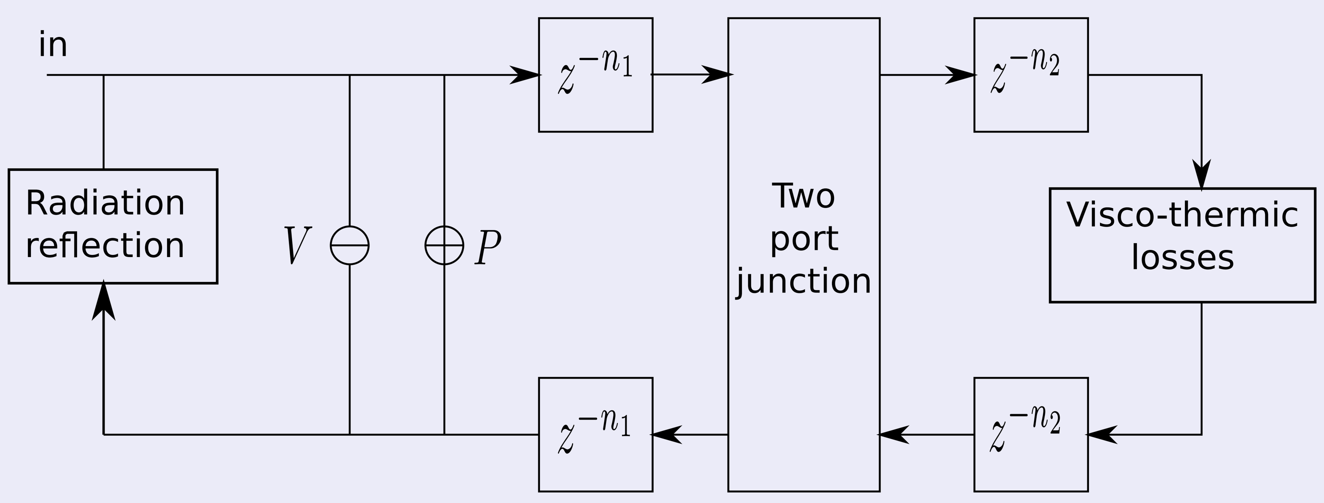


  - A ratio between low and high frequencies optimal ratios.
  - Slight inharmonicity → *ganseo*, a characteristic beating of those flutes.

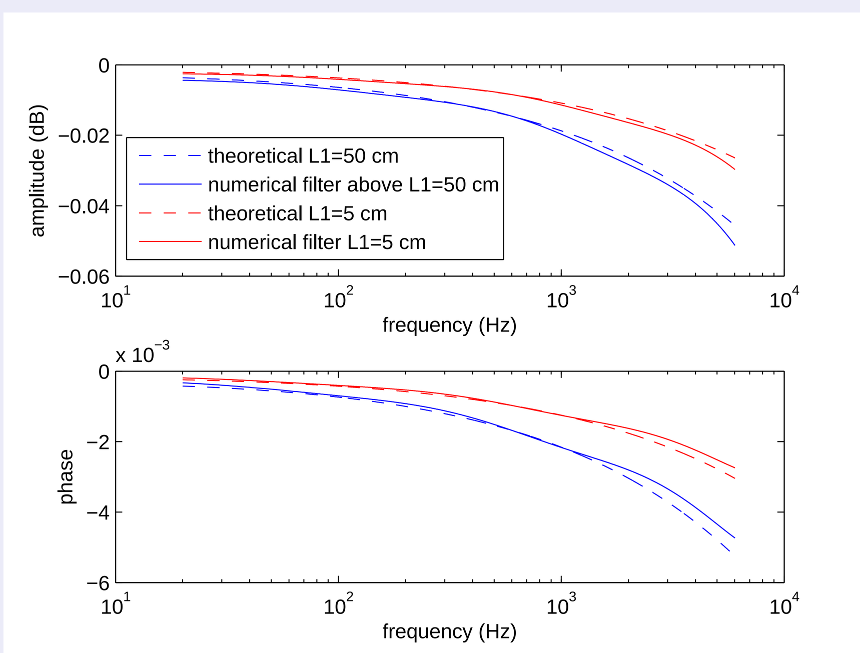
- Antaras are believed to be constructed with tools that produce fixed-diameter holes.
- Small cylinder length is adjusted with a cork.
- Impact of adjusting L_2 on the inharmonicity:



Numerical Simulation



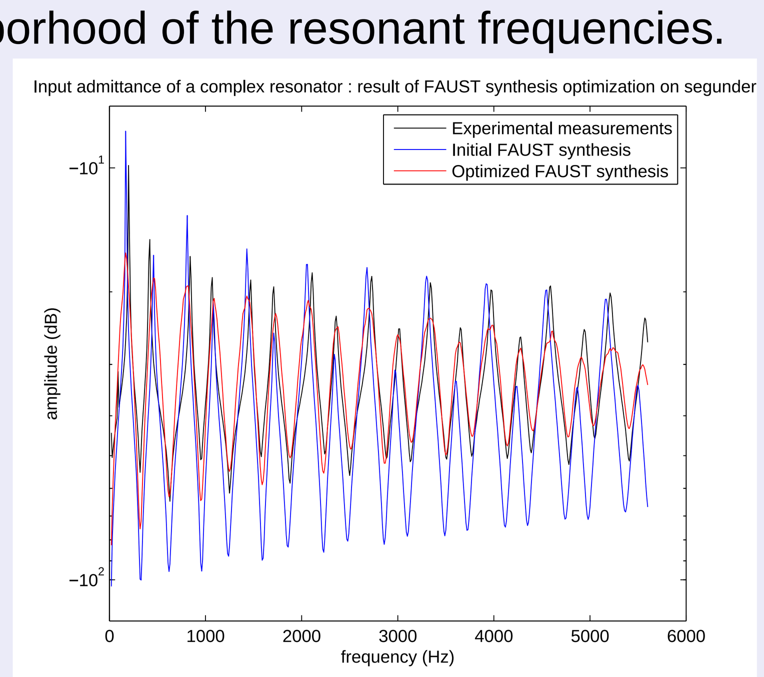
- Visco-thermal losses
  - Analogic filter with a non-linear term $\sqrt{i\omega}$.
  - Technique of fractional derivative: approximation of $\sqrt{i\omega}$ with piecewise function of slope 0 and 1 in the log-log domain.
  - Only depends on the frequency range and order of approximation: parameters dependency is kept explicit.
  - Bilinear transform to obtain a numerical filter.
- Radiation filter
  - Reflection coefficient from the impedance.
  - Approximation of derivative in the time domain: numerical filter.



- Synthesis resonator optimization
  - Synthesis admittance is measured with impulse excitation.
  - Slight error between experimental measurement and synthesis admittance (model imperfection, approximation error...).
  - The synthesis resonator explicitly depends on the physical parameters → optimization of the resonator to fit a given measurement.
  - Given Y_{exp}, find Y such that it minimizes:

$$C = (|Y_{exp}||Y - Y_{exp}|)^2.$$

  - Weight factor refines the optimization in the neighborhood of the resonant frequencies.



Conclusion

Complex resonator acoustic model

- A complete and accurate model of the two-sections resonator.
- Analysis of resonant frequencies highlight the influence of geometry on resonances.
- Better understanding of the elements to be considered when building antara-like instruments.

Physically based synthesis program

- Entirely controllable by the user.
- Sound synthesis program in FAUST and MAX patch.
- Capable of producing *sonido rajado*.
- Test-bed to study the links between physical parameters and musical aspects.
- Musically friendly interface to be used by computer-music composers.

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